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Higher-order contributions to ion-acoustic solitary waves in a multicomponent plasma consisting of warm ions and two-component nonisothermal electrons

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An integrated form of the governing equations in terms of pseudopotential higher-order nonlinear and dispersive effects is obtained by applying the reductive perturbation method for ion-acoustic solitary waves in a collisionless unmagnetized multicomponent plasma having warm ions and two-component nonisothermal electrons. The present method is advantageous because instead of solving an inhomogeneous second-order differential equation at each order, as in the standard procedure, we solve a first-order inhomogeneous equation at each order except at the lowest. The expressions of both Mach number and width of the solitary wave are obtained as a function of the amplitude of the wave for third-order nonlinear and dispersive effects. The variations of potential, width, and Mach number against soliton amplitude are shown graphically, taking into consideration the nonisothermality of two-component electrons in the plasma.

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I. INTRODUCTION

During the last few years plasma physicists have become more and more interested in studies of ion-acoustic solitons, shocks, double layers, etc. in plasmas as these have been seen to be relevant to some experimental observations and astrophysical phenomena. For the study of ion-acoustic solitary waves in a cold plasma, Washimi and Taniuti [1] were the first to derive the Korteweg-DeVries (K-dV) equation by the use of the reductive perturbation method. Later, various authors [2–9] introduced different parameters to fit the physical conditions in the plasma and obtained important results for solitons, some of which have been experimentally verified. But Jones *et al.* [10] studied both theoretically and experimentally the propagation of ion-acoustic waves in a multicomponent plasma containing ions and two types of electrons with different thermal effects. They found that the speed of the ion-acoustic wave is more strongly influenced by the low-temperature electron component than by the high-temperature component, with the degree of domination by the low-temperature component becoming extreme as the two temperatures become far apart. Goswami and Buti [11] and others [12–20] have thoroughly investigated the solitons and other aspects of ion-acoustic waves in two-electron temperature plasmas and shown the importance of cold and hot electrons in the formation of solitons and double layers, etc. However, more interesting results are found in the case of a multicomponent plasma consisting of nonisothermal electrons. In the presence of resonant electrons, the plasma behaves nonisothermally. Resonant electrons strongly interact with the wave during its evolution and therefore cannot be treated assuming the Boltzmann distribution for the electron density $n_e = \exp(\phi)$ as considered in an isothermal plasma. Schamel [21,22] first considered the nonisothermality of electrons in a plasma and showed that the electron distribution should have an expression given by Eq. (2) of his paper [22]. He found that an ion-acoustic wave in the lowest order has a sech^4 profile instead of the usual sech^2 profile. Later, Das, Paul, and Karmaker [23] and others [24,25] assumed Schamel's plasma model and investigated the effects of nonisothermality of two-temperature electrons on the formation of solitons. In this regard it is worthwhile to mention that the inclusion of ion temperature is done only in the sense of small corrections to the ion-acoustic solitons. In fact, it is found that the speed of the ion-acoustic wave is higher fro the plasma containing warm ions than for the cold ion plasma [26–28]. Moreover, it is observed experimentally by Anderson *et al.* [29] that the effect of Landau damping is very small when the ions are warm but $T_i/T_e \ll 1$.

However, from the experimental observations [30–34] it is found that theoretically predicted values of the amplitude, width, and velocity of the solitary waves do not obey the experimental results. To remove the discrepancy between the theoretical and experimental results, researchers realized the necessity of considering the higher-order nonlinear and dispersive terms in the propagation of solitary waves in a plasma. Ichikawa, Matsuhasi, and Konno [35] were the first to examine such higher-order effects by the reductive perturbation method for ion-acoustic solitons in a plasma consisting of cold ions. Considering the temperature of ions in a plasma, Lai [26] invesitgated the higher-order effect on the ion-acoustic solitons. Kodama and Taniuti [36] reconsidered this problem and have shown by the method of renormalization how to eliminate the secular terms appearing in higher-order terms of the expansion. Taking the electrons as nonisothermal, Kalita and Bujarbarua [37] investigated the propagation of higher-order ion-acoustic solitary waves in a plasma. Tagare and Reddy [38] considered the effect of higher-order nonlinearity on ion-acoustic solitary waves in a plasma consisting of negative ions and nonisothermal electrons.

It is to be noted that previous authors have only considered second-order nonlinearity and dispersive effects for the investigation of ion-acoustic solitons in the plasma. But in the present paper we investigate the effect of higher-order nonlinear and dispersive terms up to the third-order approximation on the propagation of ion-acoustic soliton waves in a plasma consisting of nonisothermal two-component electrons and warm ions. To get higher-order effects we apply the pseudopotential method [39,40] which is different from the standard method adopted in previous investigations. Recently, Das and Majumdar [28] have applied this method to investigate the effects of higher-order nonlinear and dispersive terms on the propagation of solitary waves in a plasma consisting of warm ions and isothermal electrons. Applying the reductive perturbation method, Ghosh and Das [41] investigated the higher-order contributions to the formation of solitons for a shear kinetic Alfvén wave in a low-β plasma. In this context, it is very important to mention that Watanabe and Jiang [42] developed a method for higher-order solutions of a solitary wave and obtained a fourth-order solution of a model nonlinear equation which is valid if the normalized wave amplitude is less than 0.5.

In Sec. II, we have derived the nonlinear evolution equation for ion-acoustic waves in a nonisothermal plasma having two-component electrons. To introduce the nonisothermality in the plasma, we have followed the original works of Schamel [21,22]. In Sec. III we have found the solitary wave solution with higher-order nonlinearity and dispersive effects correct up to the third-order approximations. From the first-order equation, i.e., at the lowest order, we obtain the modified K-dV (MK-dV) soliton, which has a Sech^4 profile. We have extended our calculations in the next two orders and ultimately we get the solutions of the second- and third-order equations for the ion-acoustic solitary wave. From the soliton solution up to the third-order approximation, the potential, width, and Mach number have been expressed as a function of the soliton amplitude. In Sec. IV, we analyze the results graphically for a model plasma having two-temperature electrons in nonisothermal conditions and compared these with that at the lowest order. We find that higher-order nonlinear and dispersive effects tend to increase the velocity but to decrease the width of the solitons, results which are supported by the experimental results. In Sec. V, we have summarized the present work, giving some ideas about the application and extension of our analysis for the study of ion-acoustic solitons in a plasma under different physical conditions.

II. FORMULATIONS

We consider a collisionless unmagnetized plasma consisting of ions having finite temperature T_i together with two types of nonisothermal electrons of which one is cold and the other is hot, having temperatures T_{el} and T_{eh}, respectively, under the assumption $T_i \ll T_{el}$, T_{eh}, and so Landau damping is neglected. Therefore the governing equations for such a plasma in dimensionless form are

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i u_i)=0 , \tag{1}$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}+\frac{\sigma}{n_i}\frac{\partial p_i}{\partial x}+\frac{\partial \phi}{\partial x}=0 , \tag{2}$$

$$\frac{\partial p_i}{\partial t}+u_i\frac{\partial p_i}{\partial x}+3p_i\frac{\partial u_i}{\partial x}=0 , \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_{el}+n_{eh}-n_i , \tag{4}$$

where

$$\sigma=\frac{T_i}{T_{\text{ef}}} , \quad T_{\text{ef}}=\frac{T_{el}T_{eh}}{(\mu T_{eh}+\nu T_{el})} . \tag{5}$$

n_i, n_{el}, and n_{eh} are, respectively, the number density of ions, low-, and high-temperature electrons, u_i is the ion fluid velocity, p_i is the ion fluid pressure, and ϕ is the electrostatic potential. The quantities $n_i, n_{el}, n_{eh}, u_i, p_i$, and ϕ have been made dimensionless by n_0, n_0, n_0, $(k_B T_{\text{ef}}/m_i)^{1/2}$, $n_0 k_B T_{\text{ef}}$, and $k_B T_{\text{ef}}/e$, respectively. n_0 is the equilibrium density of ions, m_i is the ion mass, and k_B is the Boltzmann constant. x and t have been made dimensionless by $(4\pi e^2 n_0/k_B T_{\text{ef}})^{-1/2}$ and $(m_i/4\pi e^2 n_0)^{1/2}$, respectively. μ and ν are the unperturbed number densities of low- and high-temperature group of electrons $(\mu+\nu=1)$, T_{ef} is the effective temperature of the plasma.

Due to the nonisothermality of two groups of electrons, the densities of cold and hot electrons can be assumed as [21,22]

$$n_{el}=\mu\left[\exp\left[\frac{T_{\text{ef}}}{T_{el}}\phi\right]\text{erfc}\left[\frac{T_{\text{ef}}}{T_{el}}\phi\right]^{1/2} +\beta_l^{-1/2}\exp\beta_l\left[\frac{T_{\text{ef}}}{T_{el}}\phi\right]\text{erfc}\left[\beta_l\frac{T_{\text{ef}}}{T_{el}}\phi\right]^{1/2}\right] , \tag{6}$$

$$n_{eh}=\nu\left[\exp\left[\frac{T_{\text{ef}}}{T_{eh}}\phi\right]\text{erfc}\left[\frac{T_{\text{ef}}}{T_{eh}}\phi\right]^{1/2} +\beta_h^{-1/2}\exp\beta_h\left[\frac{T_{\text{ef}}}{T_{eh}}\phi\right]\text{erfc}\left[\beta_h\frac{T_{\text{ef}}}{T_{eh}}\phi\right]^{1/2}\right] , \tag{7}$$

where β_l and β_h are the ratios of the number of free and trapped electrons in the low- and high-temperature groups of electrons. These parameters are expressed in the following manner:

$$\beta_l=\frac{T_{el}}{T_{elt}} , \quad \beta_h=\frac{T_{eh}}{T_{eht}} , \tag{8}$$

where T_{elt} and T_{eht} are the temperatures of the trapped electrons in the low- and high-temperature groups of electrons, and T_{el}, T_{eh} are the same for free electrons.

Expanding (6) and (7) in ascending powers of ϕ we obtain the expressions for cold and hot electrons as

$$n_{el}=\mu\left[1+\left[\frac{\phi}{\mu+\nu\beta}\right]-\frac{4}{3\pi^{1/2}}(1-\beta_l)\left[\frac{\phi}{\mu+\nu\beta}\right]^{3/2}+\frac{1}{2}\left[\frac{\phi}{\mu+\nu\beta}\right]^2-\frac{8(1-\beta_1^2)}{15\pi^{1/2}}\left[\frac{\phi}{\mu+\nu\beta}\right]^{5/2}+\frac{1}{6}\left[\frac{\phi}{\mu+\nu\beta}\right]^3+\cdots\right], \quad (9)$$

$$n_{eh}=\nu\left[1+\left[\frac{\beta\phi}{\mu+\nu\beta}\right]-\frac{4}{3\pi^{1/2}}(1-\beta_h)\left[\frac{\beta\phi}{\mu+\nu\beta}\right]^{3/2}+\frac{1}{2}\left[\frac{\beta\phi}{\mu+\nu\beta}\right]^2-\frac{8(1-\beta_h)^2}{15\pi^{1/2}}\left[\frac{\beta\phi}{\mu+\nu\beta}\right]^{5/2}+\frac{1}{6}\left[\frac{\beta\phi}{\mu+\nu\beta}\right]^3+\cdots\right], \quad (10)$$

where

$$\beta=\frac{T_{el}}{T_{eh}} . \quad (11)$$

Adding (10) and (11) we find

$$n_{el}+n_{eh}=1+\phi-\frac{4}{3}\frac{(\mu b_1+\nu b_h\beta)^{3/2}}{(\mu+\nu\beta)^{3/2}}\phi^{3/2}+\frac{1}{2}\frac{(\mu+\nu\beta^2)}{(\mu+\nu\beta)^2}\phi^2-\frac{8}{15}\frac{(\mu b_1^{(1)}+\nu b_h^{(1)}\beta^{5/2})}{(\mu+\nu\beta)^{5/2}}\phi^{5/2}+\cdots , \quad (12)$$

where

$$b_1=\frac{1-\beta_l}{\pi^{1/2}} , \quad b_h=\frac{1-\beta_h}{\pi^{1/2}} , \quad (13a)$$

$$b_l^{(1)}=\frac{1-\beta_l^2}{\pi^{1/2}} , \quad b_h^{(1)}\frac{1-\beta_h^2}{\pi^{1/2}} . \quad (13b)$$

For a solitary wave solution we assume that the dependent variables depend on a single independent variable $\xi=x-Vt$, where V is the velocity of the solitary wave. Using the boundary conditions

$$n_i\to 1 , \quad p_i\to 1 , \quad \phi\to 0 \text{ as } \xi\to\pm\infty , \quad (14)$$

we obtain from (1)–(3)

$$u_i=V\left[1-\frac{1}{n_i}\right] , \quad (15)$$

$$p_i=n_i^3 , \quad (16)$$

$$\tfrac{3}{2}\sigma n_i^4+\left[\phi-\frac{V^2}{2}-\tfrac{3}{2}\sigma\right]n_i^2+\frac{V^2}{2}=0 . \quad (17)$$

The solution of this equation for n_i^2 satisfying the requirement that $n_i=1$ at $\phi=0$ is

$$n_i^2=\frac{1}{3\sigma}[A-\phi-\{(A-\phi)^2-B^2\}^{1/2}] , \quad (18)$$

where

$$A=\frac{V^2}{2}+\frac{3}{2}\sigma , \quad B^2=3\sigma V^2 . \quad (19)$$

From Eq. (18) n_i can be expressed as

$$n_i=V[(A-\phi)+\{(A-\phi)^2+B^2\}^{1/2}]^{-1/2} . \quad (20)$$

Expanding this in ascending powers of ϕ we get

$$n_i=1+A_1\phi+A_2\phi^2+\cdots , \quad (21)$$

where

$$A_1=\frac{1}{V^3-3\sigma} , \quad A_2=6\sigma(V^2-3\sigma)^{-3}+\tfrac{3}{2}(V^2-3\sigma)^{-2} . \quad (22)$$

Substituting in (4) for n_i and $n_{el}+n_{eh}$, respectively, from (21) and (12) we get

$$\frac{d^2\phi}{d\xi^2}=\Delta_2\phi+\Delta_3\phi^{3/2}+\Delta_4\phi^2+\Delta_5\phi^{5/2}+\cdots , \quad (23)$$

where

$$\begin{aligned}\Delta_2&=1-\frac{1}{V^2-3\sigma} ,\\ \Delta_3&=-\frac{4}{3}\frac{(\mu b_l+\nu b_h\beta)^{3/2}}{(\mu+\nu\beta)^{3/2}} ,\\ \Delta_4&=\frac{1}{2}\frac{(\mu+\nu\beta^2)}{(\mu+\nu\beta)^2}-6\sigma(V^2-3\sigma)^{-3}-\tfrac{3}{2}(V^2-3\sigma)^{-2} ,\\ \Delta_5&=-\frac{8}{15}\frac{(\mu b_l^{(1)}+\nu b_h^{(1)}\beta^{5/2})}{(\mu+\nu\beta)^{5/2}} .\end{aligned} \quad (24)$$

III. SOLITARY WAVE SOLUTION WITH HIGHER-ORDER CORRECTIONS

We stretch ξ coordinates according to the relation

$$X=\epsilon^{1/4}\xi , \quad (25)$$

where ϵ is a small parameter and is a measure of the weakness of dispersion. Then Eq. (23) becomes

$$\epsilon^{1/2}\frac{d^2\phi}{dX^2}=\Delta_2\phi+\Delta_3\phi^{3/2}+\Delta_4\phi^2+\Delta_5\phi^{5/2}+\cdots . \quad (26)$$

An integral of this equation satisfying the conditions $d\phi/dX\to 0$, $\phi\to 0$ as $X\to\pm\infty$ is

$$\frac{\epsilon^{1/2}}{2}(d\phi/dX)^2=\tfrac{1}{2}\Delta_2\phi^2+\tfrac{2}{5}\Delta_3\phi^{5/2}+\tfrac{1}{3}\Delta_4\phi^3+\tfrac{2}{7}\Delta_5\phi^{7/2}+\cdots . \quad (27)$$

We now make the following perturbation expansion for ϕ and V:

$$\phi=\epsilon\phi^{(1)}+\epsilon^{3/2}\phi^{(2)}+\epsilon^{2}\phi^{(3)}+\cdots , \tag{28a}$$

$$V=V_0+\epsilon^{1/2}V^{(1)}+\epsilon V^{(2)}+\epsilon^{3/2}V^{(3)}+\cdots , \tag{28b}$$

where $V_0=(1+3\sigma)^{1/2}$, the linear velocity of the wave. With the expansion for V given by (28) the expansions for Δ_2, Δ_3, Δ_4, and Δ_5 become

$$\begin{aligned}&\Delta_2=\epsilon^{1/2}\delta_2^{(1)}+\epsilon\delta_2^{(2)}+\epsilon^{3/2}\delta_2^{(3)}+\cdots ,\\ &\Delta_3=\delta_3^{(0)} ,\\ &\Delta_4=\delta_4^{(0)}+\epsilon^{1/2}\delta_4^{(1)}+\cdots ,\\ &\Delta_5=\delta_5^{(0)} ,\end{aligned} \tag{29}$$

$$\begin{aligned}&\delta_2^{(1)}=2(1+3\sigma)^{1/2}V^{(1)} ,\\ &\delta_2^{(2)}=2(1+3\sigma)^{1/2}V^{(2)}-3(1+4\sigma)V^{(1)^2} ,\\ &\delta_2^{(3)}=2(1+3\sigma)^{1/2}V^{(3)}-6(1+4\sigma)V^{(1)}V^{(2)} ,\\ &\delta_3^{(0)}=-\frac{4}{3}\frac{(\mu b_l+\nu b_h\beta^{3/2})}{(\mu+\nu\beta)^{3/2}} ,\\ &\delta_4^{(0)}=\frac{1}{2}\frac{(\mu+\nu\beta^2)}{(\mu+\nu\beta)^2}-6\sigma-\tfrac{3}{2} ,\\ &\delta_4^{(1)}=6(1+6\sigma)(1+3\sigma)^{1/2}V^{(1)} ,\\ &\delta_5^{(0)}=-\frac{8}{15}\frac{(\mu b_l^{(1)}+\nu b_h^{(1)}\beta^{5/2})}{(\mu+\nu\beta)^{5/2}} .\end{aligned} \tag{30}$$

Substituting the expansion (28) in (27) and equating coefficients of various powers of ϵ on both sides, we get a sequence of equations for $\phi^{(i)}$'s and the equation for $\phi^{(i)}$ at each order becomes a first-order inhomogeneous differential equation for $i>1$.

A. First-order equation and its solution

In the first order, i.e., in the lowest order, which is at the order $\epsilon^{5/2}$, we get the following equation for $\phi^{(1)}$:

$$\frac{1}{2}\left[\frac{d\phi^{(1)}}{dX}\right]^2=\frac{1}{2}\delta_2^{(1)}(\phi^{(1)})^2-\tfrac{2}{5}\delta_3^{(0)}(\phi^{(1)})^{5/2} . \tag{31}$$

Setting $\phi^{(1)}=\psi^{-2}$, we get for ψ the equation

$$\frac{d\psi}{dX}=\pm\left[\frac{\delta_2^{(1)}}{4}\psi(\psi-a)\right]^{1/2} , \tag{32}$$

where $a=4\delta_3^{(0)}/5\delta_2^{(1)}$. The solution of Eq. (32) for ψ is

$$\psi=a\cosh^2\left[\frac{(\delta_2^{(1)})^{1/2}}{4}X\pm K_1\right] , \tag{33}$$

where K_1 is an arbitrary constant. This gives the following solution for $\phi^{(1)}$ that attains a maximum at $X=0$:

$$\phi^{(1)}=\lambda^2a_1\operatorname{sech}^4\eta , \tag{34}$$

where

$$a_1=\frac{25(\delta_2^{(1)})^2}{16\delta_3^{(0)}\lambda^2}=\frac{(\mu+\nu\beta)^3}{(\mu b_l+\nu b_h\beta^{3/2})^2} , \tag{35a}$$

$$\eta=(2V^{(1)}V^{(0)})^{1/2}\frac{X}{4} , \tag{35b}$$

$$\lambda=\frac{15V_0V^{(1)}}{8} ,\quad V^{(1)}=\frac{8\lambda}{15V_0} . \tag{35c}$$

The width of the soliton (D) is defined as half the value of the width of the pulse at height 0.1764 of its amplitude. Therefore we obtain from (35b), for the order $\epsilon^{1/2}$ terms, the width

$$D_1=2(15/\lambda)^{1/2} . \tag{36}$$

The Mach number correct to order $\epsilon^{1/2}$ terms of V is obtained as

$$M_1=\frac{V}{V_0}=1+\frac{8\lambda}{15V_0^2} . \tag{37}$$

B. Second-order equation and its solution

In the next order, which is at the order ϵ^3, we get the following equation for $\phi^{(2)}$:

$$\left[\frac{d\phi^{(1)}}{dX}\right]\left[\frac{d\phi^{(2)}}{dX}\right]-[\delta_2^{(1)}\phi^{(1)}-\delta_3^{(0)}(\phi^{(1)})^{3/2}]\phi^{(2)}=\tfrac{1}{2}\delta_2^{(2)}(\phi^{(1)})^2+\tfrac{1}{3}\delta_4^{(0)}(\phi^{(1)})^3 . \tag{38}$$

Differentiating (31) with respect to $\phi^{(1)}$ we get

$$\frac{d^2\phi^{(1)}}{dX^2}=\delta_2^{(1)}\phi^{(1)}-\delta_3^{(0)}(\phi^{(1)})^{3/2} . \tag{39}$$

By the use of this relation Eq. (38) can be expressed as follows, where the independent variable X has been replaced by η given by (35b):

$$\frac{d\phi^{(2)}}{d\eta}-\frac{\phi_{\eta\eta}^{(1)}}{\phi_\eta^{(1)}}\phi^{(2)}=\frac{8\delta_2^{(2)}}{\delta_2^{(1)}}\frac{(\phi^{(1)})^2}{\phi_\eta^{(1)}}+\frac{16\delta_4^{(0)}}{3\delta_2^{(1)}}\frac{(\phi^{(1)})^3}{\phi_\eta^{(1)}} . \tag{40}$$

Multiplying both sides of this equation by the integrating factor $1/\phi_\eta^{(1)}$ and then integrating we get the following solution for $\phi^{(2)}$:

$$\phi^{(2)}=-\frac{2a_1\lambda^2\delta_2^{(2)}}{\delta_2^{(1)}}\eta\tanh\eta\operatorname{sech}^4\eta+\frac{2a_1\lambda^2\delta_2^{(2)}}{\delta_2^{(1)}}\operatorname{sech}^4\eta+\frac{4a_1^2\lambda^4\delta_4^{(0)}}{3\delta_2^{(1)}}(1+\tanh^2\eta)\operatorname{sech}^4\eta+K_2\operatorname{sech}^4\eta\tanh\eta , \tag{41}$$

where K_2 is an arbitrary constant.

For a uniformly valid expansion of ϕ the ratio $\phi^{(2)}/\phi^{(1)}$ should remain finite at $\eta\to\pm\infty$. This demands that the coefficient of the first term on the right hand side of (41) must be equal to zero. Therefore we have

$$\delta_2^{(2)}=0 \tag{42}$$

which gives

$$V^{(2)}=\frac{3}{2}\frac{(1+4\sigma)}{V_0}(V^{(1)})^2=\frac{32}{75}\frac{(1+4\sigma)}{V_0^3}\lambda^2 . \tag{43}$$

Consequently the expression for $\phi^{(2)}$ given by (41) becomes

$$\phi^{(2)}=\left[\frac{-4a_1^2\lambda^4\delta_4^{(0)}}{3\delta_2^{(1)}}(1+\tanh^2\eta)+K_2\tanh\eta\right]\text{sech}^4\eta\ . \tag{44}$$

For $\phi^{(2)}$ to attain maximum at $\eta=0$ we must set $K_2=0$ and therefore the final expression for $\phi^{(2)}$ becomes

$$\phi^{(2)}=\lambda^3 a_2(1+\tanh^2\eta)\text{sech}^4\eta\ , \tag{45}$$

where

$$a_2=\frac{5(\mu+\nu\beta)^6}{8(\mu b_l+\nu b_h\beta^{3/2})^4}\left[\frac{(\mu+\nu\beta^2)}{(\mu+\nu\beta)^2}-12\sigma-3\right]. \tag{46}$$

The solitary wave solution up to second order is obtained by using (34) and (45) in (28a) and is given by

$$\Phi_2=\phi^{(1)}+\phi^{(2)}$$
$$=\alpha_{02}[\alpha_{12}+\alpha_{22}\tanh^2\eta]\text{sech}^4\eta\ , \tag{47}$$

where $\alpha_{02}=\lambda^2(a_1+a_2\lambda)$,

$$\alpha_{12}=1\ ,\qquad \alpha_{22}=\frac{a_2\lambda}{(a_1+a_2\lambda)}\ . \tag{48}$$

The Mach number M_2 correct up to $V^{(2)}$ is obtained by using (35c) and (43) in (28b) and is given by

$$M_2=1+\frac{8}{15V_0^2}+\frac{32(1+4\sigma)\lambda^2}{75V_0^4}\ . \tag{49}$$

Moreover, from (35b) the width of the soliton is obtained as

$$D_2=2(15/\lambda)^{1/2}\eta_{D_2}(\lambda)\ , \tag{50}$$

where $\eta_{D_2(\lambda)}$ is the positive root of η of the following equation:

$$\gamma_3\tanh^6\eta+\gamma_2\tanh^4\eta+\gamma_1\tanh^2\eta+\gamma_0=0\ , \tag{51}$$

where γ_3, γ_2, γ_1, and γ_0 are denoted by

$$\gamma_3=\alpha_{22}\ ,\quad \gamma_2=1-2\alpha_{22}\ ,\quad \gamma_1=\alpha_{22}-2\ ,\quad \gamma_0=0.8236\ . \tag{52}$$

C. Third-order equation and its solution

The equation at the third order, which is at the order $\epsilon^{7/2}$, can be put in the following form by the use of the relation (39):

$$\frac{d}{d\eta}\left[\frac{\phi^{(3)}}{\phi_\eta^{(1)}}\right]=8\frac{(\phi^{(2)})^2}{(\phi_\eta^{(1)})^2}+\frac{8\delta_2^{(3)}}{\delta_2^{(1)}}\frac{(\phi^{(1)})^2}{(\phi_\eta^{(1)})^2}-\frac{12\delta_3^{(0)}}{\delta_2^{(1)}}\frac{(\phi^{(2)})^2(\phi^{(1)})^{1/2}}{(\phi_\eta^{(1)})^2}+\frac{16\delta_4^{(0)}}{\delta_2^{(1)}}\frac{\phi^{(2)}(\phi^{(1)})^2}{(\phi_\eta^{(1)})^2}+\frac{16\delta_4^{(1)}}{3\delta_2^{(1)}}\frac{(\phi^{(1)})^3}{(\phi_\eta^{(1)})^2}-\frac{32\delta_5^{(0)}}{7\delta_2^{(1)}}\frac{(\phi^{(1)})^{7/2}}{(\phi_\eta^{(1)})^2}-\frac{1}{2}\frac{(\phi_\eta^{(2)})^2}{(\phi_\eta^{(1)})^2}\ . \tag{53}$$

Integrating this equation we get the following solution for $\phi^{(3)}$:

$$\phi^{(3)}=-\frac{2a_1\lambda^2\delta_2^{(3)}}{\delta_2^{(1)}}\eta\,\text{sech}^4\eta\tanh\eta+\left[\left[\lambda^4a_3+\frac{2a_1\lambda^2\delta_2^{(3)}}{\delta_2^{(1)}}\right]+\lambda^4b_3\tanh^2\eta+\lambda^4c_3\tanh^4\eta\right]\text{sech}^4\eta-4a_1\lambda^2K_3\tanh\eta\,\text{sech}^4\eta\ , \tag{54}$$

where

$$a_3=\frac{140a_1^3\lambda^2(\delta_4^{(0)})^2}{(9\delta_2^{(1)})^2}+\frac{4a_1^2\delta_4^{(1)}}{3\delta_2^{(1)}}-\frac{8a_1^{5/2}\lambda\delta_5^{(0)}}{7\delta_2^{(1)}}\ ,$$
$$b_3=-\frac{208a_1^3\lambda^2(\delta_4^{(0)})^2}{9(\delta_2^{(1)})^2}+\frac{4a_1^2\delta_4^{(1)}}{3\delta_2^{(1)}}-\frac{16a_1^{5/2}\lambda\delta_5^{(0)}}{7\delta_2^{(1)}}\ , \tag{55}$$
$$c_3=-\frac{28a_1^3\lambda^2(\delta_4^{(0)})^2}{9(\delta_2^{(1)})^2}+\frac{8a_1^{5/2}\lambda\delta_5^{(0)}}{21\delta_2^{(1)}}\ ,$$

and K_3 is an arbitrary constant.

Now for a uniformly valid expansion of ϕ the ratio $\phi^{(3)}/\phi^{(2)}$ must remain finite at $\eta\to\pm\infty$. Therefore the coefficient of the first term on the right hand side of (54) must vanish and consequently we have

$$\delta_2^{(3)}=0\ . \tag{56}$$

This equation gives the following third-order correction to the velocity of propagation of the solitary wave:

$$V^{(3)}=\frac{256(1+4\sigma)^2}{375V_0^5}\lambda^3\ . \tag{57}$$

Since $\delta_2^{(3)}=0$, the expression for $\phi^{(3)}$ given by (54) becomes

$$\phi^{(3)}=[\lambda^4a_3+\lambda^4b_3\tanh^2\eta+\lambda^4c_3\tanh^4\eta-4a_1\lambda^2K_3\tanh\eta]\text{sech}^4\eta\ . \tag{58}$$

For $\phi^{(3)}$ to attain maximum at $\eta=0$ we must set $K_3=0$ and therefore the final expression for $\phi^{(3)}$ becomes

$$\phi^{(3)}=\lambda^4(a_3+b_3\tanh^2\eta+c_3\tanh^4\eta)\operatorname{sech}^4\eta \ , \tag{59}$$

where a_3, b_3, and c_3 given by (55) can be expressed as

$$a_3=\frac{875}{256}\bar{a}+4\bar{b}+\frac{4}{7}\bar{c} \ ,$$

$$b_3=-\frac{325}{64}\bar{a}+4\bar{b}+\frac{8}{7}\bar{c} \ ,$$

$$c_3=-\frac{175}{256}\bar{a}-\frac{4}{21}\bar{c} \ ,$$

where $\bar{a}, \bar{b}$, and $\bar{c}$ are given by

$$\bar{a}=\left[\frac{\mu+\nu\beta^2}{(\mu+\nu\beta)^2}-12\sigma-3\right]^2\frac{(\mu+\nu\beta)^9}{(\mu b_1+\nu b_h\beta^{3/2})^2} \ , \tag{60a}$$

$$\bar{b}=\frac{(1+6\sigma)(\mu+\nu\beta)^6}{(\mu b_l+\nu b_h\beta^{3/2})^4} \ , \tag{60b}$$

$$\bar{c}=\frac{(\mu+\nu\beta)^5(\mu b_l^{(1)}+\nu b_h^{(1)}\beta^{5/2})}{(\mu b_l+\nu b_h\beta^{3/2})^5} \ . \tag{60c}$$

To obtain the solitary wave solution up to the third-order approximation we substitute the values of $\phi^{(1)},\phi^{(2)}$, and $\phi^{(3)}$ given by (34), (45), and (59), respectively, in the expression for ϕ given by (28a) and we ultimately get

$$\phi_3=\phi^{(1)}+\phi^{(2)}+\phi^{(3)}$$
$$=\alpha_{03}[\alpha_{13}+\alpha_{23}\tanh^2\eta+\alpha_{43}\tanh^4\eta]\operatorname{sech}^4\eta \ , \tag{61}$$

where

$$\alpha_{03}=\lambda^2(a_1+a_2\lambda+a_3\lambda^2) \ , \tag{62a}$$

$$\alpha_{13}=1 \ , \tag{62b}$$

$$\alpha_{23}=\frac{\lambda(a_2+b_3\lambda)}{a_1+a_2\lambda+a_3\lambda^2} \ , \tag{62c}$$

$$\alpha_{43}=\frac{c_3\lambda^2}{a_1+a_2\lambda+a_3\lambda^2} \ . \tag{62d}$$

This α_{03} is the amplitude of the solitary wave. To find the Mach number M correct to order $\epsilon^{3/2}$ terms, we substitute in (28b) for $V^{(1)}$, $V^{(2)}$, and $V^{(3)}$ given, respectively, by (35c), (43), and (57) and obtain

$$M_3=\frac{V}{V_0}=1+\frac{8\lambda}{15V_0^2}+\frac{32(1+4\sigma)\lambda^2}{75V_0^4}+\frac{256(1+4\sigma)^2\lambda^3}{375V_0^6} \ . \tag{63}$$

The width of the soliton D_3 is obtained from (35b),

$$D_3=2(15/\lambda)^{1/2}\eta_{D_3}(\lambda) \ . \tag{64}$$

The positive root of $\eta_{D_3}(\lambda)$ is obtained from the following equation of η:

$$\gamma_4'\tanh^8\eta+\gamma_3'\tanh^6\eta+\gamma_2'\tanh^4\eta+\gamma_1'\tanh^2\eta+\gamma_0'=0 \ , \tag{65}$$

where

$$\gamma_4'=\alpha_{43}, \quad \gamma_3'=-2\alpha_{43}+\alpha_{23} \ ,$$
$$\gamma_2'=\alpha_{43}-2\alpha_{23}+1, \quad \gamma_1'=\alpha_{23}-2 \ , \tag{66}$$
$$\gamma_0'=0.8236=\gamma_0 \ .$$

It is to be noted that the expressions for soliton widths as a function of soliton amplitudes may be obtained by eliminating the parameter λ between (36) and (35a), (50) and (48a), and (64) and (62a) for the first-order, second-order, and third-order solitons, respectively. Next eliminating λ between (37) and (55a), (49) and (48a), and (63) and (62a), we get the expressions for Mach numbers as a function of soliton amplitudes.

IV. RESULTS AND DISCUSSIONS

From the above analysis we see that the expression (34) is the well-known first-order MK-dV soliton solution which was obtained by Das, Paul, and Karmaker [18] and others. The expressions (45) and (59) are the second-order and third-order MK-dV soliton solutions. Previously authors obtained a MK-dV (second-order) solution similar to our present result given by (45). But none of the previous authors obtained the MK-dV soliton considering the third-order corrections. Our expression (53) gives the soliton solution up to the third-order contribution of the nonlinear and dispersive effect in a plasma. It is evident from the above expressions that nonisothermality and two-component electrons have significant contributions to the formation of solitons in the plasma. It is interesting to note that the third-order soliton solution ϕ_3 given by (53) reduces to the second-order soliton solution ϕ_2 given by (47), if the terms containing the third-order nonlinear and dispersive effect are neglected, i.e., $a_3=b_3=c_3=0$. Similarly, the second-order solution ϕ_2 given by (47) reduces to the first-order solution ϕ_1 given by (34), if we put $a_2=0$.

To investigate the characteristics of the solitary waves, numerical estimations are made considering a plasma having $\mu=0.15$, $\nu=0.85$, $b_l=0.15$, $b_h=0.40$, $\beta=0.025$, $\sigma=0.02$, $b_l^{(1)}=0.260$, and $b_h^{(1)}=0.5164$, and results have been compared for the first-order, second-order, and third-order solitons. In fact, two-component electrons are found in cathode discharge tubes [43] and in double plasma (DP) machines [44]. In space and thermonuclear plasmas, two distinct Maxwellian electron populations with different temperatures have also been found [45,46]. Using the above data for a multicomponent plasma consisting of warm ions and two-component nonisothermal electrons, the potentials ϕ_1, ϕ_2, and ϕ_3 given by (34), (47), and (61), respectively, are calculated and then plotted in Fig. 1. It is seen that the third-order potential is greater than the second-order potential which is also greater than the first-order potential. If the ions are assumed to be cold, i.e., $\sigma=0$, these potentials become smaller than that of the warm ionic plasma. It is also observed that the

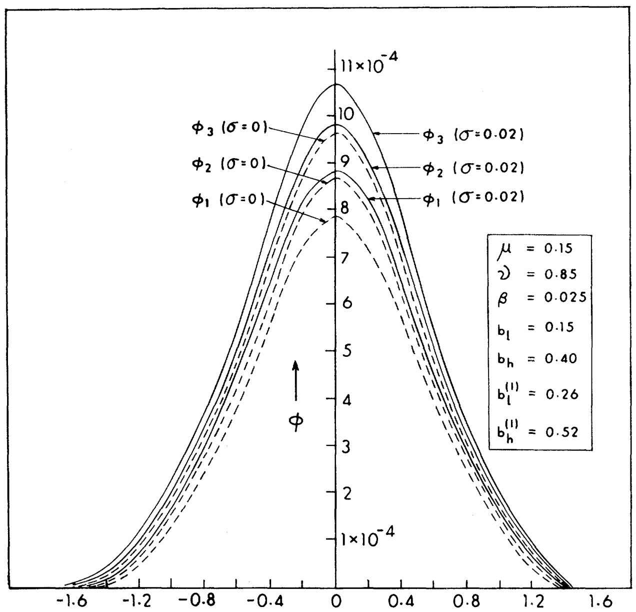

FIG. 1. Steady ion-acoustic soliton solutions in nonisothermal two-component electron plasma. ...... for cold ions ($\sigma=0$). —— for warm ions ($\sigma=0002$).

temperature of the ions only plays a role in the sense of small corrections to the solitary wave but the nature of the potentials remains the same.

From (36), (50), and (64), the widths D_1, D_2, and D_3 for the first-order, second-order, and third-order solitons are numerically calculated and then plotted against the amplitude of the soliton wave (Fig. 2). It is observed that the width decreases with the increase of the amplitude and higher-order nonlinear and dispersive effects play a role to increase the width for a given amplitude.

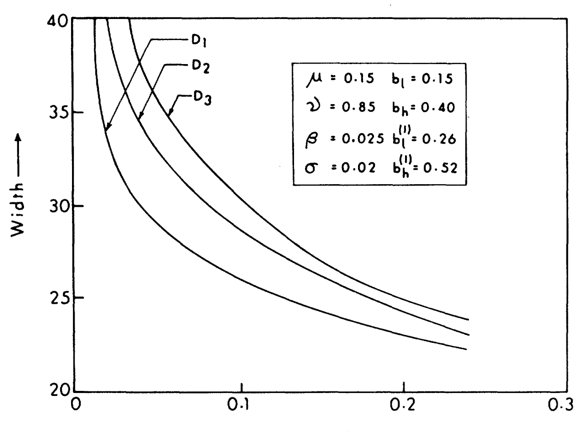

FIG. 2. Dependence of widths on the amplitudes of ion-acoustic solutions.

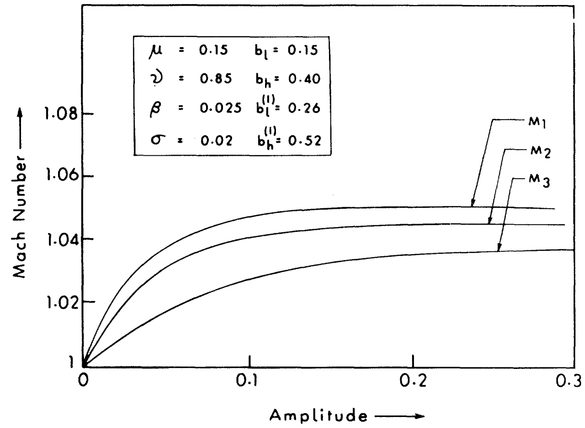

FIG. 3. Dependence of Mach numbers on the amplitudes of ion-acoustic solitons.

The Mach numbers M_1, M_2, and M_3 are numerically estimated from (37), (49), and (63) for the plasma having the parameters stated above. The Mach numbers are then plotted in Fig. 3. It is seen that the Mach number increases with increase of the amplitude and also by the higher-order nonlinear and dispersive effects.

It is to be noted that various authors [30–34] have experimentally studied the behavior of ion-acoustic solitons in plasmas. However, we have not yet seen any experimental report for the ion-acoustic soliton in a plasma having warm ions and two-temperature nonisothermal electrons. So our present results cannot be compared with the experimental observations.

V. CONCLUDING REMARKS

We have investigated the contributions of higher-order nonlinear and dispersive effects on ion-acoustic solitary waves in a plasma consisting of warm ions and two groups of nonisothermal electrons. In the lowest order the solitary wave has a sech^4 profile. We have carried out the calculations up to the next two higher orders, i.e., up to second and third order. Considering higher-order corrections, the expressions for both the Mach number and the width of the solitary wave are obtained as a function of its amplitude. In our present analysis, the reductive perturbation method is applied to an integrated form of the governing equations in terms of the pseudopotential. The advantage of this method is that, instead of solving an inhomogeneous second-order differential equation at each order as in the standard procedure, we are to solve a first-order inhomogeneous differential equation at each order except at the lowest order.

In recent years, a relativistic effect has been considered for the investigation of ion-acoustic solitary waves in a plasma [47]. It has been found that the presence of streaming ions and the relativistic effect have a contribution to the formation of solitons, shocks, and double layers in the plasma [48–55]. Higher-order soliton solutions, up to second-order approximations, only are obtained by using the standard mathematical technique. Following our present method we may find the second-order and also third-order solutions for the ion-acoustic solitary waves in a simpler way, which would help to investigate the role of the relativistic effect as well as higher-order contributions of nonlinearity and dispersiveness on the solitary wave in unmagnetized or magnetized plasma.

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